AIAA JOURNAL
Vol. 40, No. 9, September 2002

Efficient Aerodynamic Design Method Using a Tightly Coupled Algorithm

Chun-ho Sung*
Rutgers University, Piscataway, New Jersey 08854
and
Jang Hyuk Kwon†
Korea Advanced Institute of Science and Technology, Taejon 305-701, Republic of Korea

An efficient aerodynamic design optimization method for the compressible Euler equations is presented. A gradient-based optimization method is used to find the optimal design, in conjunction with a continuous adjoint sensitivity analysis method. To improve the efficiency of the design method, a tightly coupled algorithm based on a step-size estimation is proposed. The efficiency of the present method is demonstrated through drag minimizations of a wing and an airfoil. The result shows that the cost of the present method is significantly lower than that of the loosely coupled algorithm.

Nomenclature

C_i	=	flux Jacobian matrix in the computational domain
d	=	design variables vector
F_i	=	flux vector in the computational domain
f_i	=	flux vector in the physical domain
$\mathcal{G}$	=	gradient vector of the object function
I	=	object function
q	=	conservative flow variables vector
R	=	residual vector in the computational domain
$\mathcal{S}$	=	searching direction
ϵ	=	step size along the searching direction $\mathcal{S}$
ψ	=	Lagrangian multiplier vector

Subscript

face	=	value at cell face

Superscript

T	=	transpose of vector or matrix

Introduction

AERODYNAMIC design optimization (ADO) has been an important research area for the past decade.[1-5] It provides an automated design procedure with the help of numerical optimization and computational fluid dynamics (CFD). In ADO, CFD replaces the wind-tunnel test of a model geometry, and numerical optimization indicates how the model should be changed to improve the aerodynamic performance. By virtue of ADO, the turn around time and cost of design can be substantially reduced.[6] In spite of these advantages, ADO has not been widely used for practical design problems due to a huge amount of computing time.[4,7]

Early work on ADO was limited by the number of design variables. The finite difference method was used to find the design sensitivity; hence, flow solutions should be recalculated in proportion to the number of design variables.[1] An alternative of the finite difference method is the adjoint method, which has been introduced by Jameson[8] and Pironneau[9] to fluid dynamics problems. In the adjoint method, the sensitivity can be found from the solution of partial differential equations, namely, adjoint equations, regardless of the number of design variables. Thus, the computing time of sensitivity analysis can be significantly reduced. The adjoint method has been successfully applied for several three-dimensional problems, including the design optimization of a full aircraft.[4,7,10]

Even with the adjoint method, the computing time of design optimization is still in the order of a hundred times of flow analysis.[11,12] A reason for this large cost is mainly due to the coupling level of optimization and flow analysis. In a conventional aerodynamic design optimization algorithm, the flow, sensitivity andoptimization are independently solved in a row.[13] Hereinafter, this method will be referred as a loosely coupled algorithm. The cost of a loosely coupled algorithm can be estimated as follows. To find the sensitivity of the design problem, the flow and adjoint equations are fully recalculated at every design iteration. After sensitivity analysis, a new design can be found through a one-dimensional line search. Because the aerodynamic design optimization problem is a nonlinear one, an iterative process or quadratic interpolation is used for the line search. Consequently, it requires more than three solutions of flow equations. When it is assumed that the cost of the adjoint equations is the same as that of the flow equations, the cost of one design cycle can be estimated as, at least, five times the flow analysis. If the optimum design can be found in 10 design iterations, the total cost of design optimization can be estimated to be as much as 50 flow analyses.

Efforts have been made to improve the efficiency of a loosely coupled algorithm.[14-17] Most of these works are based on the idea that the coupling level of the design optimization can be very essential to computing cost. These methods will be referred as a tightly coupled algorithm. Ta'asan et al. suggested the one-shot method, which uses a multigrid method for solving flow and adjoint equations and the optimization problem simultaneously.[15] The pseudotime method of Iollo et al. shares the same idea with the one-shot method, although it used a pseudo-time-marching method for design optimization.[16] Feng and Pulliam suggested an all-at-once method by using the reduced Hessian sequential quadratic programming method.[17] These efforts showed the cost of design optimization can be reduced to less than 10 times the amount of computing time for the flow solution. However, these algorithms are only applied for simple one-dimensional nozzles or airfoil shape optimization.

The main purpose of this paper is to develop an efficient ADO method that is fast enough for practical three-dimensional applications. We use a gradient-based optimization, an adjoint method for sensitivity analysis, and a tightly coupled algorithm. The equations for the design variables, namely, design equations, are derived from

Received 8 June 2000; revision received 5 April 2002; accepted for publication 8 April 2002.

*Postdoctoral Research Associate, Department of Mechanical and Aerospace Engineering, 98 Brett Road. Member AIAA.

†Associate Professor, Department of Aerospace Engineering, 373-1 Kusong-Dong, Yusong-Gu. Senior Member AIAA.

the optimality condition. The design variables are updated by a time-marching method during the design optimization. The three sets of equations, flow, adjoint, and design equations, are simultaneously solved by a design iteration cycle. One design iteration consists of several time steppings for flow and adjoint equations and one time stepping of design equations. A multigrid diagonalized alternating direction implicit (DADI) method[18,19] is used for the time stepping of both the flow and the adjoint equations. The design equations are solved by a simple explicit time stepping, and the step size is determined by a simple estimation formula of Ta'asan et al.[15] The resulting design procedure is similar to that of the one-shot method[15] except that the multigrid method is not applied for design variables. The flow and adjoint solutions are not fully converged, but only several multigrid cycles are used during design iterations. This strategy can result in a substantial reduction of computing cost of design optimization. The three-dimensional compressible Euler equations are used for flow analysis, and the second-order upwind total variation diminishing (TVD) scheme is used for discretization.

To show the efficiency of the presented method, drag minimization problems for an airfoil and three-dimensional wing under transonic flow conditions are presented. The efficiency of the proposed design method is assessed by the cost ratio of design and flow analysis and comparison with a cost of the loosely coupled algorithm.

Governing Equations

The three-dimensional compressible Euler equations in Cartesian coordinates x_1, x_2, and x_3 can be written in the conservation form as

$$\frac{\partial \boldsymbol{q}}{\partial t} + \frac{\partial \boldsymbol{f}_i}{\partial x_i} = 0 \tag{1}$$

where

$$\boldsymbol{q} = \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{bmatrix}, \qquad \boldsymbol{f}_i = \begin{bmatrix} \rho u_i \\ \rho u_i u_1 + p\delta_{i1} \\ \rho u_i u_2 + p\delta_{i2} \\ \rho u_i u_3 + p\delta_{i3} \\ \rho u_i H \end{bmatrix} \tag{2}$$

and ρ is the density, u_i is the velocity component for the x_i direction, and p, E, and H are pressure, total energy, and total enthalpy, respectively. The repeated index represents the tensor summation convention. The pressure is determined by the equation of state

$$p = (\gamma - 1)\rho\left(E - \tfrac{1}{2}u_i u_i\right) \tag{3}$$

and the total enthalpy is

$$H = E + p/\rho \tag{4}$$

where $\gamma = 1.4$ for air and is the ratio of specific heats. For convenience, the physical coordinates system will be transformed to the computational coordinates ξ_1, ξ_2, and ξ_3.

In the computational domain, the Euler equations yield

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial \boldsymbol{F}_i}{\partial \xi_i} = 0 \tag{5}$$

where

$$\boldsymbol{Q} = \frac{1}{J}\boldsymbol{q}, \qquad \boldsymbol{F}_i = \frac{1}{J}\left(\frac{\partial \xi_i}{\partial x_j}\boldsymbol{f}_j\right)$$

and J is the transformation Jacobian.

Adjoint Equations

The cost function of aerodynamic design can be written as a function of flow variables and geometric properties:

$$I = I(\boldsymbol{q}, \boldsymbol{d}) \tag{6}$$

where I is the cost function, $\boldsymbol{q}$ is the vector of flow variables, and $\boldsymbol{d}$ is a vector of design variables that defines the target geometry of the optimization problem. Because the governing equations for the flowfield can be incorporated in the optimization problem as constraints, the design sensitivity can be derived from the variational method or the optimal control theory.

For the steady state, the flowfield governing equations $\boldsymbol{R}$ can be expressed as

$$\boldsymbol{R}(\boldsymbol{q}, \boldsymbol{d}) = 0 \tag{7}$$

When the Lagrange multiplier ψ is introduced, the augmented cost function or Lagrangian becomes

$$I = I + \psi^T \boldsymbol{R} \tag{8}$$

and its first variation can be written as

$$\delta I = \left\{\frac{\partial I^T}{\partial \boldsymbol{q}} + \psi^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{q}}\right\}\delta \boldsymbol{q} + \left\{\frac{\partial I^T}{\partial \boldsymbol{d}} + \psi^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{d}}\right\}\delta \boldsymbol{d} \tag{9}$$

When the multiplier ψ satisfies the adjoint equation

$$\frac{\partial I^T}{\partial \boldsymbol{q}} + \psi^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{q}} = 0 \tag{10}$$

the first term in Eq. (9) vanishes, and the change in the cost function δI can be obtained from $\delta \boldsymbol{d}$.

Adjoint Formulation for Euler Equations

Equation (10) differs in form according to the governing equations. In this section, the derivation of continuous adjoint equations for the Euler equations is briefy described. The description of detailed derivations may be found in the literature.[3] The cost function of present work is limited to the drag coefficient, and the constraint is the lift coefficient. They can be expressed with pressures on the body surface p and the shape of body h, which is a function of design variables $\boldsymbol{d}$.

With the transformation to the computational coordinates ξ_1, ξ_2, and ξ_3, the body surface can be transformed onto a simple region in the computational domain. We assume that the wing surface is transformed onto the $\xi_2 = 0$ surface. Then, the cost function I can be written as

$$I = \int_{B_S} h(\boldsymbol{d}) g(p)\, \mathrm{d}\xi_j \tag{11}$$

where B_S is the domain of body surface and $g(p)$ is the lift or drag coefficients or a blended function of both. When the Lagrange multiplier ψ is introduced, the variation of cost function δI can be calculated without the variation of pressure δp. The full details of the derivations may be found in Refs. 3 and 20.

By use of the continuous adjoint method, the adjoint equations for the Euler equations derived from Eq. (5) can be written as

$$\boldsymbol{C}_i^T \frac{\partial \psi}{\partial \xi_i} = 0 \tag{12}$$

where

$$\boldsymbol{C}_i = \frac{\partial \boldsymbol{F}_i}{\partial \boldsymbol{Q}} \tag{13}$$

Equation (12) is similar to the linearized form of the Euler equation except that the flux Jacobian matrix is transposed. Therefore, it can be solved by the same numerical procedure as for the Euler equations, if appropriate boundary conditions are imposed.

The boundary conditions for the adjoint equations at the body surface are determined as follows[3]:

$$\psi_2 \frac{1}{J}\frac{\partial \xi_2}{\partial x_1} + \psi_3 \frac{1}{J}\frac{\partial \xi_2}{\partial x_2} + \psi_4 \frac{1}{J}\frac{\partial \xi_2}{\partial x_3} = h\frac{\mathrm{d}g}{\mathrm{d}p} \tag{14}$$

where ψ_2, ψ_3, and ψ_4 are the second to fourth components of the Lagrange multiplier ψ. Equation (14) eliminates the contribution of pressure variation δp in the variation of cost function δI. Finally, the variation of cost function becomes

$$\delta I = \int_{B_S} g\delta h \, \mathrm{d}\xi_j - \int_D \psi^T \left\{ \frac{\partial}{\partial \xi_i} \left[\delta \left(\frac{1}{J} \frac{\partial \xi_i}{\partial x_j} \right) \boldsymbol{f}_j \right] \right\} \mathrm{d}\xi_k \tag{15}$$

where D is the computational domain. The variations of metric $\delta[(1/J)(\partial \xi_i/\partial x_j)]$ can be found by the finite difference method (FDM). Therefore, the gradient of costfunction $\mathcal{G}$ is

$$\mathcal{G} = \delta I / \delta \boldsymbol{d} \tag{16}$$

Numerical Method

Both the Euler equations and adjoint equations are discretized using the cell-centered finite volume method. The numerical fluxes at the cell faces are calculated by Roe's[21] approximate Riemann solver together with the second-order upwind TVD scheme (see Ref. 22).

The semidiscrete form of Eq. (5) can be obtained via finite volume discretization. When integrated cellwise in the computational space domain, the governing equations yield

$$\frac{\mathrm{d}}{\mathrm{d}t}(S\boldsymbol{Q})_{ijk} + \boldsymbol{R}_{ijk}(\boldsymbol{Q}) = 0 \tag{17}$$

where

$$\boldsymbol{R}_{ijk} = \boldsymbol{F}_{1i+\frac{1}{2},j,k} - \boldsymbol{F}_{1i-\frac{1}{2},j,k} + \boldsymbol{F}_{2i,j+\frac{1}{2},k} - \boldsymbol{F}_{2i,j-\frac{1}{2},k} + \boldsymbol{F}_{3i,j,k+\frac{1}{2}} - \boldsymbol{F}_{3i,j,k-\frac{1}{2}} \tag{18}$$

where S_{ij} is the cell volume and $\boldsymbol{F}_i$ are the numerical fluxes at the cell interfaces. The numerical fluxes of the second-order TVD scheme are constructed as follows:

$$\boldsymbol{F}_{\text{face}} = \tfrac{1}{2}[\boldsymbol{F}_L + \boldsymbol{F}_R - (|\tilde{A}|\Delta \boldsymbol{Q} - \boldsymbol{L})_{\text{face}}] \tag{19}$$

where subscripts L and R indicate left and right states of each cell face, the tilde is Roe's average[21] value, and $\boldsymbol{L}$ is the antidiffusive flux of the second-order upwind TVD scheme.[22] The van Leer limiter is used for satisfying TVD condition, and Harten's entropy fix function is introduced to prevent nonphysical solutions.

The discretization procedure for the adjoint equations are straightforward because the two sets of equations are of the same form. Although the adjoint equations are linear, an iterative method is used for steady-state solution. Because the numerical fluxes for the adjoint equations are very crucial for the accuracy of computed design sensitivity, a consistent flux function derived from the discrete Euler equations is used.[23]

The steady-state solution is obtained via the DADI algorithm and the multigrid method with a modified sawtooth cycle.[18,19] The Riemann invariants are used for Euler equations at the far-field boundary and the flow tangency condition at the body surface. For the adjoint equations, the Lagrangian multipliers are kept zero at the far-field boundary, and Eq. (14) is used at the body surface.

The solution procedure is parallelized using the domain decomposition method and message passing interface library.[24] The resulting parallel code is a single instruction multiple data type.

Design Algorithm

Shape Function and Grid Modification

The wing geometry is described with the initial shape and its perturbations. The shape functions in aerodynamic design are the basis of perturbation functions. Because the design result will be highly influenced by the shape functions, one must choose proper functions according to design goals. In this work, the Hicks–Henne function[25] is used as the shape function. It has been used by many researchers for airfoil and wing design problems.[1,3] When the Hicks–Henne function is used, the modified section profile can be written as

$$y(x) = y^0(x) + \sum d_i h_i(x) \tag{20}$$

where y^0 is the initial geometry, d_i are design variables, and h_i are Hicks–Henne functions, which are defined as

$$h_i(x) = [\sin(\pi x^{t_1})]^{t_2} \tag{21}$$

where t_1 locates the maximum of the bump and t_2 determines the width of the bump.

For the two-dimensional problem, 10 Hicks–Henne[25] functions are used. These design variables are only used to modify the upper surface of airfoil. The lower surface is kept constant during the design optimization. For the three-dimensional problem, five sections of the wing are selected for the basis section. At each basis section, 10 Hicks–Henne functions are used for modifying the section shape. The first five variables are applied on the upper surface and another five on the lower surface. Other sections of the wing are linearly interpolated from the neighboring basis sections.

After the surface is modified, the grid system should be also modified according to the wing surface. Because the displacement of surface is usually very small, a simple arc-length-based linear interpolation is sufficient for the grid modification. The new grid system $\boldsymbol{x}^{n+1}$ is calculated as

$$\boldsymbol{x}_{ijk}^{n+1} = \boldsymbol{x}_{ijk}^0 + c_{ijk}(\boldsymbol{x}^{n+1} - \boldsymbol{x}^0)_{i1k} \tag{22}$$

where

$$c_{ijk} = \frac{\sum_{j=1}^{j-1} |\boldsymbol{x}_{i,j+1,k} - \boldsymbol{x}_{ijk}|}{\sum_{j=1}^{j\,\max-1} |\boldsymbol{x}_{i,j+1,k} - \boldsymbol{x}_{ijk}|} \tag{23}$$

and $|\cdot|$ indicates the $L2$ norm.

Tightly Coupled Algorithm

The loosely coupled algorithms iteratively solve the flow and adjoint equations during the optimization. In these method, the optimization problem is fully separated from the flow solution procedure, and several flow calculations are required at every design iteration including line search.[13,15] As mentioned in the Introduction, this strategy results in a substantial increase of computing time.

In this paper, a tightly coupled algorithm is proposed based on the design equations. Two necessary conditions of the constraint optimization problem are feasibility and optimality.[26] The flowfield governing equations can be feasibility conditions of aerodynamic design optimization. They ensure that the flow variables are satisfying physical laws. The optimality condition defines the gradient of the cost function at the optimum.

One interesting viewpoint of the constraint optimization was introduced by Iollo et al.,[16] where the optimization procedure is regarded as a pseudo-time marching in the design space. If we use a loosely coupled algorithm, the design optimization is only performed in the feasible region of design space. However, the feasibility of the design point during iterations is not necessary, as far as it can be guaranteed at the optimum. This means that the flow solutions should not necessarily be converged during the design optimization. An alternative cost-effective algorithm can be constructed, by relaxing the feasibility condition during the design iterations.

Basically, a new tightly coupled algorithm is based on pseudo-time marching of the design variables. By the use of the optimality condition, the gradient of object function should be zero at optimum. Therefore, the following optimality condition can be referred as design equations:

$$\mathcal{G}(\boldsymbol{d}) = 0 \tag{24}$$

In a loosely coupled algorithm, Eq. (24) is solved by a nonlinear optimization method. During the optimization, the flow and adjoint equations are repeatedly solved to provide the value of object function and sensitivity. In present method, a global design cycle is used to simultaneously solve the flow, adjoint, and design equations.

The design equations can be also solved by a time-marching method. By the introduction of a pseudotime t^* in the design space, the design equations can be written as follows:

$$\frac{\partial \boldsymbol{d}}{\partial t^*} + \mathcal{G} = 0 \tag{25}$$

Then, the design variables at new time step $n+1$ can be found by an explicit method:

$$\Delta \boldsymbol{d}/\Delta t^* + \mathcal{G}^n = 0 \tag{26}$$

or

$$d^{n+1} = d^n - \Delta t^* \mathcal{G} \tag{27}$$

where Δt^* is a time step for the design equations. Note that the discretized design equations (27) are the same as that of the steepest descent method.[26]

Now, we have three sets of equations, flow, adjoint, and design equations. A global design cycle is constructed with the time-marching procedures of the three sets of equations. If the design should be feasible during the optimization, the flow equations need to be fully converged in a design cycle. Theoretically, it is possible to construct the design cycle just with one time integration of flow, adjoint, and design equations. However, the three sets of equations have different numerical characteristics; thus, it is not effective to construct such a cycle. For the three-dimensional case, the present design cycle consists of 20 multigrid cycles of flow and adjoint equations, and one time integration of design equations.

The present algorithm can be referred as a tightly coupled algorithm in the sense that the flow, adjoint, and design equations are simultaneously solved by a global design cycle. The flow and adjoint equations are marching to steady state during the design iterations, while the design variables are also updated in the same iterations.

On the other hand, the time step of the design equations should be carefully chosen to maintain the stability of the global design cycle. The stability analysis for the design equations can give the limits of the time step Δt^*; however, it is not an easy task when the structure of $\mathcal{G}$ cannot be defined as an analytic form.

An alternative is to determine Δt^* by using the analogy with the unconstrained optimization procedure.[15] For an unconstrained optimization problem, the new design variables are determined by the searching direction $\mathcal{S}$ and the step size ϵ. With the steepest descent method, the design variable at the $n+1$th step will be updated as

$$\boldsymbol{d}_{n+1} = \boldsymbol{d}_n + \epsilon(\mathcal{S}/|\mathcal{S}|) \tag{28}$$

where the search direction $\mathcal{S}$ is defined as $\mathcal{S} = -\mathcal{G}$. The step size ϵ is expected to minimize $|\mathcal{G}(d+n+1)|$. That is,

$$\frac{\partial \mathcal{G}(\boldsymbol{d}_{n+1})}{\partial \epsilon} = \frac{\partial \mathcal{G}(\boldsymbol{d}_n + \epsilon \mathcal{S})}{\partial \epsilon} = 0 \tag{29}$$

The Taylor expansion of Eq. (29) yields (see Ref. 15)

$$\epsilon = -\frac{\mathcal{G}^T \nabla_d \mathcal{G} \mathcal{S}}{\mathcal{S}^T (\nabla_d \mathcal{G})^T \nabla_d \mathcal{G} \mathcal{S}} \tag{30}$$

where $\nabla_d \mathcal{G}$ is a Hessian matrix. The production of Hessian matrix and searching direction $\nabla_d \mathcal{G} \mathcal{S}$ is calculated from the finite difference with the previous design point:

$$\nabla_d \mathcal{G} \mathcal{S} = [\mathcal{G}(d + \bar{\epsilon}\mathcal{S}) - \mathcal{G}(d)]/\bar{\epsilon} \tag{31}$$

From Eqs. (28) and (30), Δt^* can be written as

$$\Delta t^* = \epsilon/|\mathcal{G}| \tag{32}$$

Finally, the present design procedure can be summarized as follows:

1) Apply multigrid cycles for flow and adjoint equations.
2) Calculate grid sensitivity by finite difference.
3) Evaluate $\mathcal{G}$, gradient of cost function, by the use of Eq. (16).
4) Calculate Δt^* by Eq. (32).
5) Update design variables using Eq. (27).
6) Repeat steps 1–5 until $\mathcal{G} = 0$.

Numerical Results

The present design method is applied to drag minimization problems of a two-dimensional airfoil and a three-dimensional wing.

The cost function for the aerodynamic design is a penalty function form to keep the lift coefficient constant. The same cost function is used by Anderson and Venkatakrishnan,[27] which can be written as

$$I = \tfrac{1}{2}(C_L - C_{L0})^2 + \tfrac{10}{2} C_D^2 \tag{33}$$

where C_{L0} is the initial lift coefficient.

Two convergence criteria for the design optimization are used: One is that the $L2$ norm of the gradient vector is less than 10^{-4} normalized by its initial value, and the other is the relative change of cost function is less than 10^{-4}.

Two-Dimensional Airfoil Optimizations

The first test case is a drag minimization of a Royal Aircraft Establishment (RAE) 2822 airfoil. The flow condition is Mach number 0.73 at an angle of attack of 2.79 deg. The test case has been used in a previous study to validate design algorithms.[1,12] Under this flow condition, a strong shock wave appears on the upper surface of the airfoil.

The optimizations are performed with a loosely coupled algorithm using both the finite difference and adjoint sensitivity methods, as well as the proposed tightly coupled algorithm. For the loosely coupled algorithm, the Broyden–Fletcher–Goldfarb–Shanno method is used to determine the searching direction, and the step size is determined by a one-dimensional line search with a sequential quadratic interpolation. The computational grid system is O type with 129 grid points. For the loosely coupled algorithm, the solutions at the previous design point are used as the initial solutions of subsequent designs.

All three design methods are converged by the second convergence criterion, that is, the relative change of the cost function is less than 10^{-4}. The loosely coupled algorithm with the finite difference sensitivity takes 9 design iterations, whereas that with adjoint sensitivity is converged after 10 iterations. The tightly coupled algorithm converges after 30 design iterations.

Figure 1 shows the pressure contours around an airfoil before and after the design by the tightly coupled algorithm. The strong shock wave on the airfoil is removed by the design. The design optimization results are shown in Fig. 2. The lift and drag coefficients of designed airfoils are also given in Table 1. The lift coefficients of the airfoils are identical, whereas a small difference can be seen in the drag coefficients. The surface pressure distributions are very similar except at the leading edge. The result of the tightly coupled method is slightly different from the other methods. However, all three design optimization methods end up with the shock-free airfoil. The designed airfoils are almost identical, as can be seen in Fig. 2b.

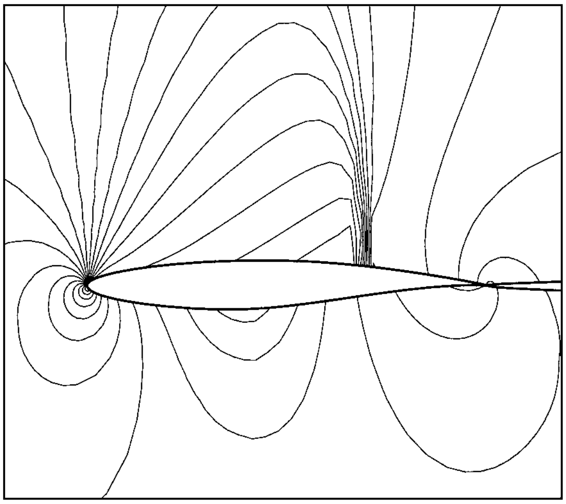
In the two-dimensional airfoil optimization problem, the design results of the loosely and tightly coupled methods are essentially the same. However, the computing costs for the design optimizations are much different. Table 2 shows the computing cost of the design optimization algorithms. The costs are given by the relative cost of a flow analysis when the $L2$ norm of the density is less than 10^{-6}. The computing cost of the design optimization can be dramatically reduced by using the tightly coupled algorithm. The relative

Table 1 Aerodynamics coefficients of design results

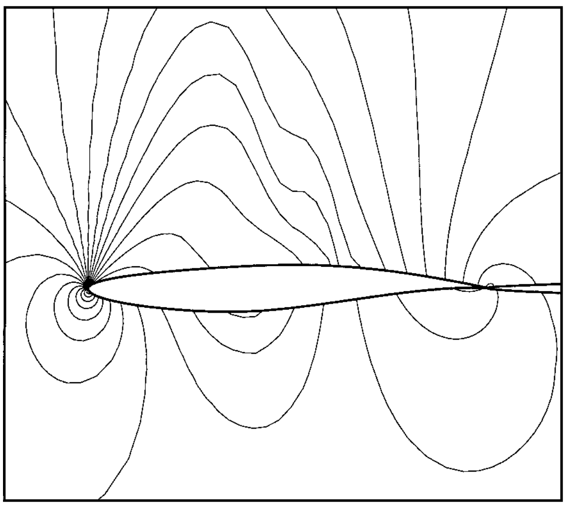
Design algorithm	C_L	C_D
Loosely coupled (FDM)	99.6	58.0
Loosely coupled (adjoint)	99.6	58.0
Tightly coupled (adjoint)	99.6	60.6

Table 2 Computing costs of design optimization algorithms

Design algorithm	Number of iterations	CPU, s	Cost
Loosely coupled (FDM)	9	4130	193.9
Loosely coupled (adjoint)	10	1897	89.1
Tightly coupled (adjoint)	30	170	8.0

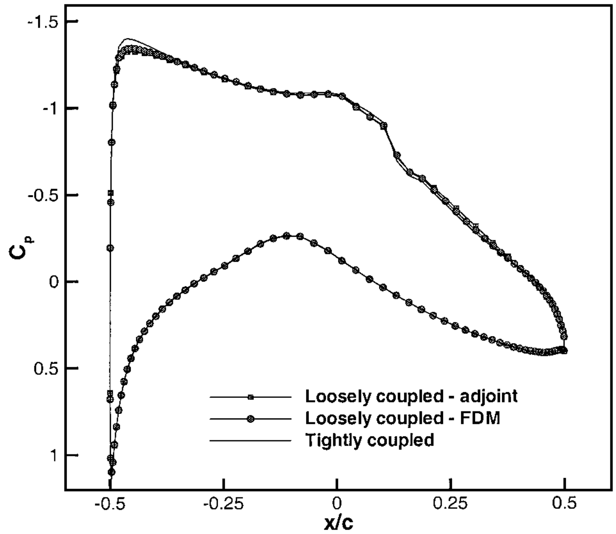


RAE 2822

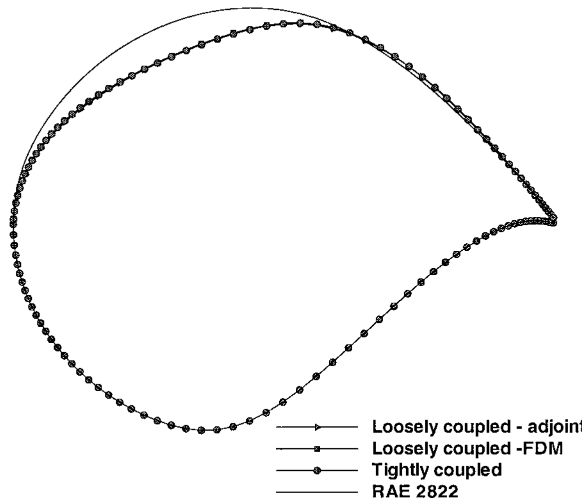


Designed airfoil

Fig. 1 Pressure contours around airfoil before and after design.



a) Surface pressure distributions



b) RAE 2822 and designed airfoils

Fig. 2 Results of design optimization of various methods.

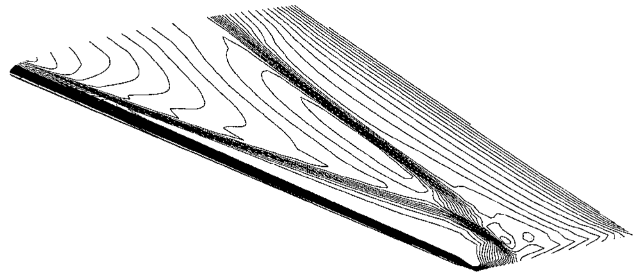
cost of the present tightly coupled algorithm is just eight times that of a flow analysis. The efficiencies of tightly and loosely coupled algorithms are different by a factor of 10, even with the adjoint sensitivity method. Because we solve two sets of field equations, that is, flow and adjoint, the ideal cost ratio is expected to 2.0. The present cost ratio is larger than the ideal value; nonetheless, it is very low compared with the conventional loosely coupled algorithm.

Three-Dimensional Wing Optimization

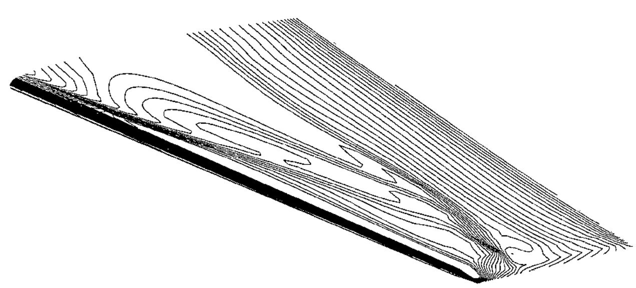
The initial geometry for the three-dimensional wing optimization is the ONERA M6 wing, and the flow condition is Mach number 0.84 at an angle of attack of 3.06 deg. The object function of the design optimization is the same as that of the two-dimensional case. The computational grid system is O–H type with 129×33 grid points. A four-level modified sawtooth cycle is used for the multigrid, and initial solutions are accelerated by the mesh-sequencing method. The Cray T3E parallel computer, at the supercomputing center in the Republic of Korea, is used for numerical calculations, and the test cases are conducted with 16 processors.

The design was stopped by the second condition of the convergence criteria. The number of design iterations is 20, and total computing time is 995 s for each processor. The number of multigrid cycles for the flow equations is 236, during the design iterations, and that for the adjoint equations is 379.

Figure 3 shows comparison of pressure contours on the wing surface between the initial and the designed wings. The strong λ shock on the upper surface of the wing is smeared by the design. Figure 4 shows pressure plots on the selected wing spans. The shock



ONERA M6



Designed wing

Fig. 3 Pressure contours on the wing surface before and after design.

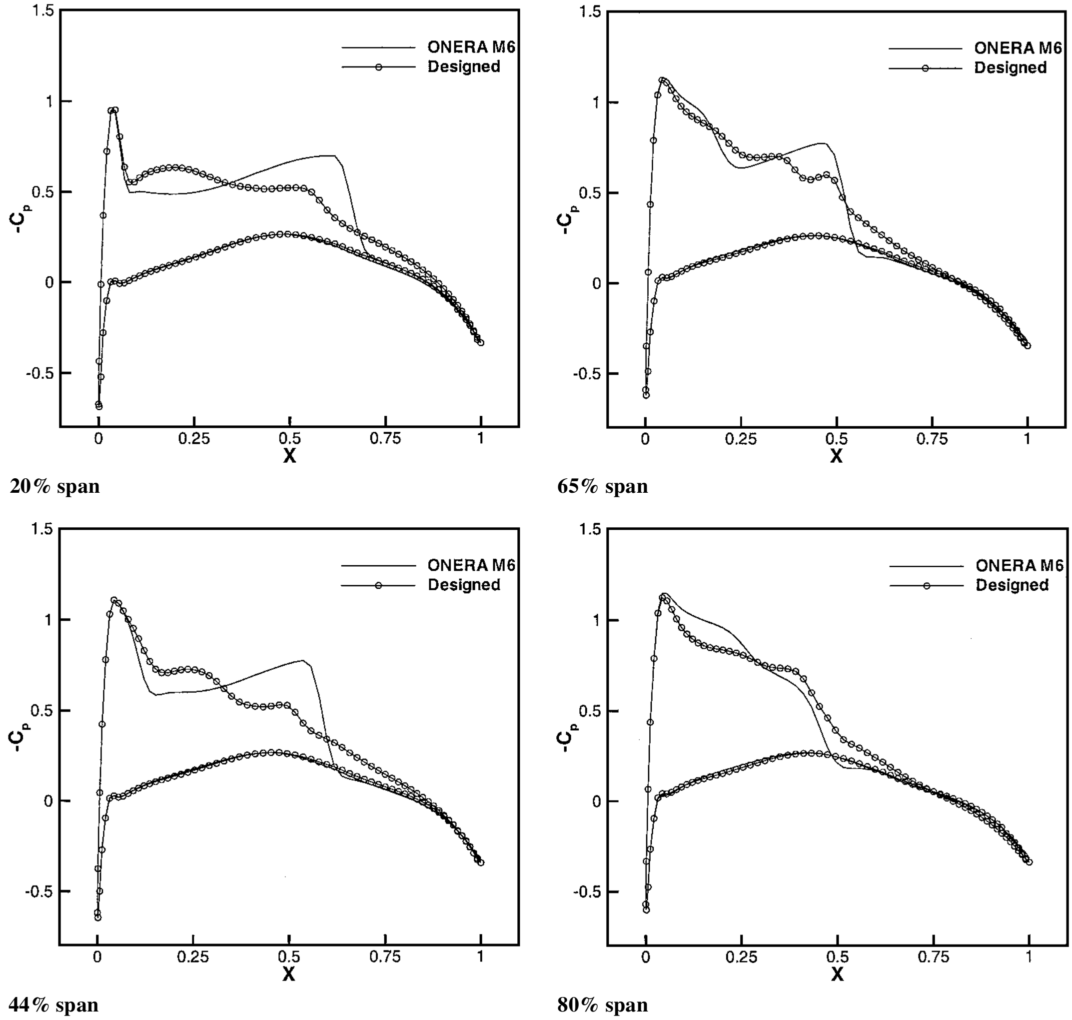


Fig. 4 Comparisons of pressure distributions on selected wing sections.

Fig. 5 Geometries of the ONERA M6 and designed wing.

waves are smeared but not fully removed. One possible reason is the shape function. In the present design, only five selected wing sections are modified by the design variables, and other parts of the wing surface are interpolated from the selected sections. Thus, the exibility of shape function is somewhat limited, and it can result in the remaining shock waves.

Figure 5 shows the shape change of the wing by the design optimization. The lower surface of the designed wing is similar to that of ONERA M6. The upper surface of the designed wing is modified to reduce the shock strength. The wing becomes slender at the wing root, and it becomes thicker along the wing span.

Figure 6 shows the history of the $L2$ norm of the gradient and cost function, and Fig. 7 shows the histories of the lift and drag coefficients during design iterations. The wave drag is reduced by 20%, and the lift is maintained at 98% of initial value. Both the $L2$ norm and the cost function decrease monotonically during the design cycles, but flatten out later.

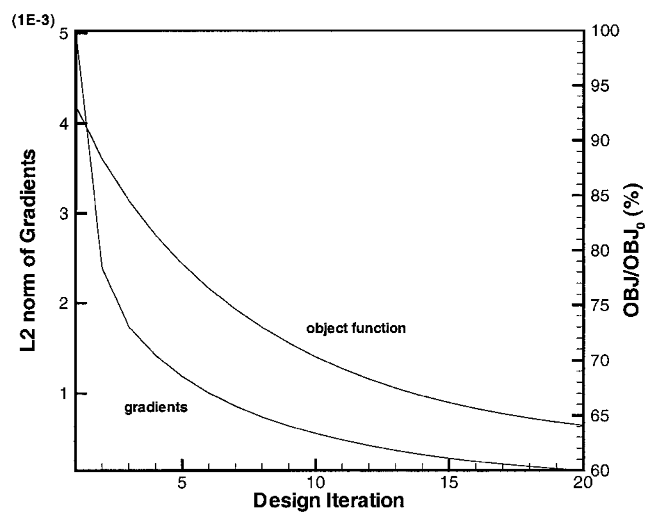


Fig. 6 Histories of the cost function and $L2$ norm of gradient during design iterations.

The total cost of the design optimization is again compared with that of flow analysis alone. Because the computing time of the flow analysis significantly varies according to the convergence criterion, several criteria are chosen for comparison. The cost ratio of design and flow analysis is about 7.65 when the convergence criterion of flow analysis is 10^{-6} and 12.6 for 10^{-5}. Even with 10^{-4} of the convergence criterion, the relative cost is 22.4.

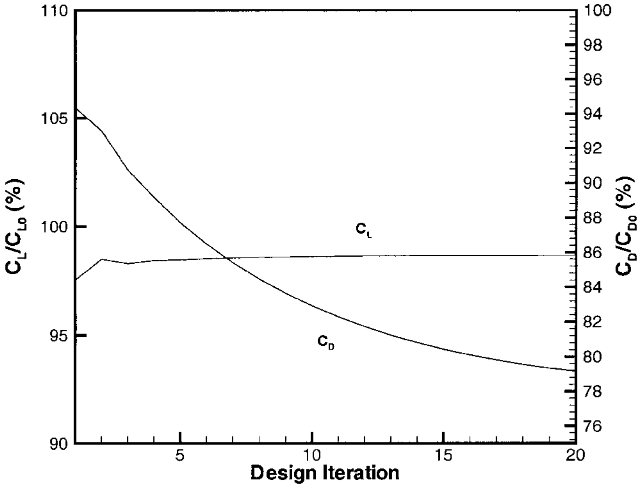


Fig. 7 Histories of the lift and drag coefficient.

Conclusions

An ADO method based on a tightly coupled algorithm is presented. The present method uses the adjoint method for sensitivity analysis, which is adequate for large-scale design problems. The design equations are derived from the optimality condition, and they are solved by a time-marching method, together with the flow and adjoint equations. The step size for the design equations is determined by a simple relation without an expensive line search algorithm.

Two and three-dimensional inviscid drag minimization problems are solved by using the present design algorithm. The present design algorithm successfully removes or smears the shock waves in the flowfield, and it reduces a significant amount of wave drag. In terms of relative cost to the flow analysis, the present design algorithm takes 8.0 and 7.65 for two- and three-dimensional design optimization problem, respectively. The substantial reduction in the computing cost can be observed, compared with the conventional loosely coupled algorithm. For two-dimensional test case, the cost for the proposed tightly coupled algorithm is less than 10% of the loosely coupled one.

References

[1]Lee, K. D., and Eyi, S., "Aerodynamic Design via Optimization," International Council of the Aeronautical Sciences, ICAS Paper 90-6.9.1, 1990.

[2]Baysal, O., and Eleshaky, M., "Aerodynamic Design Optimization Using Sensitivity Analysis and Computational Fluid Dynamics," AIAA Paper 91-0471, 1991.

[3]Reuther, J., and Jameson, A., "Control Theory Based Airfoil Design Using the Euler Equations," AIAA Paper 94-4272, 1994.

[4]Elliott, J., and Peraire, J., "Practical Three-Dimensional Aerodynamic Design and Optimization Using Unstructured Grids," AIAA Paper 96-4170, 1996.

[5]Anderson, W., and Bonhaus, D., "Aerodynamic Design on Unstructured Grids for Turbulent Flows," NASA TM 112867, 1997.

[6]Jameson, A., "Reengineering the Design Process Through Computation," *Journal of Aircraft*, Vol. 36, No. 1, 1999, pp. 36–50.

[7]Burgreen, G., and Baysal, O., "Three-Dimensional Aerodynamic Shape Optimization Using Discrete Sensitivity Analysis," *AIAA Journal*, Vol. 34, No. 9, 1996, pp. 1761–1770.

[8]Jameson, A., "Aerodynamic Design via Control Theory," *Journal of Scientific Computing*, Vol. 3, Nov. 1988, pp. 233–260.

[9]Pironneau, O., "On Optimum Design in Fluid Mechanics," *Journal of Fluid Mechanics*, Vol. 64, 1974, pp. 97–111.

[10]Reuther, J., Jameson, A., Alonso, J., Rimlinger, M., and Saunders, D., "Constrained Multipoint Aerodynamic Shape Optimization Using an Adjoint Formulation and Parallel Computers," *Journal of Aircraft*, Vol. 36, No. 1, 1999, pp. 51–74.

[11]Frank, P., and Shubin, G., "A Comparison of Optimization-Based Approaches for a Model Computational Aerodynamics Design Problem," *Journal of Computational Physics*, Vol. 98, No. 1, 1992, pp. 74–89.

[12]Sung, C., and Kwon, J. H., "Aerodynamic Design Optimization Using Adjoint Method," *Journal of the Korean Society for Aeronautical and Space Sciences*, Vol. 26, No. 8, 1998, pp. 1–10.

[13]Newman, P. A., Hou, G. J.-W., and Taylor, A. C., III, "Observations Regarding Use of Advanced CFD Analysis, Sensitivity Analysis, and Design Codes in CFD," Inst. for Computer Applications in Science and Engineering, ICASE Rept. 96-16, Hampton, VA, 1996.

[14]Rizk, M., "Single-Cycle Scheme: A New Approach to Numerical Optimization," *AIAA Journal*, Vol. 21, No. 12, 1983, pp. 1640–1647.

[15]Ta'asan, S., Kuruvila, G., and Salas, M., "Aerodynamic Design and Optimization in One Shot," AIAA Paper 92-0025, 1992.

[16]Iollo, A., Salas, M., and Ta'asan, S., "Shape Optimization Governed by the Euler Equations Using an Adjoint Method," Inst. for Computer Applications in Science and Engineering, ICASE Rept. 93-78, Hampton, VA, 1993.

[17]Feng, D., and Pulliam, T., "An All-at-Once Reduced Hessian SQP Scheme for Aerodynamics Design Optimization," Research Inst. for Advanced Computer Science, TR 95-19, Moffett Field, CA, 1995.

[18]Pulliam, T., and Chaussee, D., "A Diagonal Form of an Implicit Approximate-Factorization Algorithm," *Journal of Computational Physics*, Vol. 39, Feb. 1981, pp. 347–363.

[19]Sung, C., Park, S.-H., and Kwon, J. H., "Multigrid Diagonalized-Alternating Direction Implicit Method for Compressible Flows," AIAA Paper 2001-2556, 2001.

[20]Reuther, J., "Aerodynamic Shape Optimization Using Control Theory," Ph.D. Dissertation, Univ. of California, Davis, CA, 1996.

[21]Roe, P., "Approximate Riemann Solver, Parameter Vectors and Difference Schemes," *Journal of Computational Physics*, Vol. 43, No. 2, 1981, pp. 357–372.

[22]Park, T. S., and Kwon, J. H., "An Improved Multistage Time Stepping for Second-Order Upwind TVD Schemes," *Computers and Fluids*, Vol. 25, No. 7, 1996, pp. 629–645.

[23]Sung, C., and Kwon, J. H., "Accurate Aerodynamic Sensitivity Analysis Using Adjoint Equations," *AIAA Journal*, Vol. 38, No. 2, 2000, pp. 243–250.

[24]Snir, M., Otto, S., Huss-Lederman, S., Walker, D., and Dongarra, J., *MPI: The Complete Reference*, MIT Press, Cambridge, MA, 1996.

[25]Hicks, R., and Henne, P., "Wing Design by Numerical Optimization," *Journal of Aircraft*, Vol. 15, No. 7, 1978, pp. 407–412.

[26]Luenberger, D., *Linear and Nonlinear Programming*, Addison Wesley, Reading, MA, 1989.

[27]Anderson, W., and Venkatakrishnan, V., "Aerodynamic Design Optimization on Unstructured Grids with a Continuous Adjoint Formulation," AIAA Paper 97-0643, 1997.

A. Chattopadhyay
Associate Editor